

①

Let $p = p(x, b) = \frac{e^{cx} - 1}{e^{cb} - 1}$, where $c = \frac{-2\mu}{\sigma^2}$ ①

$w = w(x, b, L) = (1-L)(1-p)(-x) + p(b-x)$
 $w = -x + px + Lx - Lpx + pb - px$
 $w = -x + Lx - Lpx + pb$ ②

Note $\frac{\partial p}{\partial x} = \frac{c e^{cx}}{e^{cb} - 1}$, $\frac{\partial p}{\partial b} = \frac{-(e^{cx} - 1) c e^{cb}}{(e^{cb} - 1)^2}$ ③

Taking partial derivatives:

$\frac{\partial w}{\partial x} = -1 + L - L \frac{\partial p}{\partial x} \cdot x - Lp + \frac{\partial p}{\partial x} \cdot b$
 $= L - 1 - Lp - Lx \frac{\partial p}{\partial x} + b \frac{\partial p}{\partial x}$
 $= L - 1 - L \left(\frac{e^{cx} - 1}{e^{cb} - 1} \right) - Lx \left(\frac{c e^{cx}}{e^{cb} - 1} \right) + b \left(\frac{c e^{cx}}{e^{cb} - 1} \right)$

So $\frac{\partial w}{\partial x} = 0$ gives

$(L-1)(e^{cb} - 1) - L(e^{cx} - 1) - Lx c e^{cx} + b c e^{cx} = 0$
 $L e^{cb} - L - L e^{cx} + L - Lx c e^{cx} + b c e^{cx} = 0$
 $L e^{cb} - L e^{cx} - Lx c e^{cx} + b c e^{cx} = 0$

$L e^{cb} - L e^{cx} - Lx c e^{cx} + b c e^{cx} = 0$ ④

(2)

$$\frac{\partial w}{\partial b} = -L \frac{\partial p}{\partial b} x + \frac{\partial p}{\partial b} \cdot b + p \quad (\text{from (2)})$$

$$\text{So } \frac{\partial w}{\partial b} = 0 \Rightarrow -L \frac{\partial p}{\partial b} x + \frac{\partial p}{\partial b} b + p = 0$$

$$\Rightarrow \frac{\partial p}{\partial b} = \frac{p}{(Lx - b)} \quad (5)$$

From (1) + (3) + (5)

$$\frac{(-1)(e^{cx} - 1)ce^{cb}}{(e^{cb} - 1)^2} = \frac{p}{(Lx - b)} = \frac{(e^{cx} - 1)}{(e^{cb} - 1) \cdot (Lx - b)}$$

$$\frac{(-1)ce^{cb}}{(e^{cb} - 1)} = \frac{1}{(Lx - b)}$$

$$(-1)ce^{cb}(Lx - b) = e^{cb} - 1$$

$$-cLxe^{cb} + bce^{cb} - e^{cb} + 1 = 0 \quad (6)$$

Subtracting equation (6) from (4)

$$Le^{cb} - Le^{cx} - Lxc e^{cx} + bce^{cx} + cLxe^{cb} - bce^{cb} = 0$$

$$e^{cb}(L + cLx - bc) + e^{cx}(-L - Lxc + bc) = 0$$

$$e^{cb}(L + cLx - bc) = e^{cx}(L + Lxc - bc) \quad (7)$$

(3)

if $L + cLx - bc \neq 0$ then $e^{cb} = e^{cx}$, ~~so $b=x$~~
 So $b=x$. This corresponds to not playing and
 is the degenerate local maximum.

Otherwise, assume $L + cLx - bc = 0$ (8)

From equation #6, $e^{cb}(-cLx + bc - 1) = -1$

pluggin in $-cLx + bc = L$, we get

$$e^{cb}(L-1) = -1$$

$$e^{cb} = \frac{1}{1-L}$$

$$cb = \ln\left(\frac{1}{1-L}\right)$$

$$b = \frac{1}{c} \cdot \ln\left(\frac{1}{1-L}\right) \quad (9) //$$

Then from (8), $x = \frac{bc - L}{cL}$ (10) //

$$b = \frac{1}{c} (-1) \ln(1-L)$$

$$b = \frac{\sigma^2}{2\mu} \ln(1-L) \quad (11) //$$

(9)

$$\begin{aligned}
 X &= \frac{bc - L}{cL} = \frac{(-1)\ln(1-L) - L}{cL} \\
 &= \frac{\ln(1-L) + L}{-cL} \\
 &= \frac{\sigma^2}{2\mu} \left(1 + \frac{\ln(1-L)}{L} \right)
 \end{aligned}$$

$$X = \frac{\sigma^2}{2\mu} \left(1 + \frac{\ln(1-L)}{L} \right)$$

$$b - X = \frac{-\sigma^2}{2\mu} - \frac{\sigma^2}{2\mu} \cdot \frac{\ln(1-L)}{L} + \frac{\sigma^2}{2\mu} \ln(1-L)$$

$$b - X = \frac{\sigma^2}{2\mu} \left(\ln(1-L) - \frac{\ln(1-L)}{L} - 1 \right)$$